Introduction to Probability Theory.
Warm-up before the first midterm.

Problem 1. We select at random ten different cards from a standard deck of 52 cards. Find the probability that

(i) we will choose exactly 2 clubs, 3 spades, and one ace.
(ii) we will choose exactly 2 clubs, 3 spades, and at most one ace.

Problem 2. The first urn contains 3 white and 2 black balls, the second contains 4 white balls. In the first round we choose one urn (say, by flipping a symmetric coin), pick up one ball from it, and put it into the other urn. Then in the second round we again choose one urn by flipping a coin, and select a ball from it. What is the probability that the ball we chose in the first round was black provided we know that the ball chosen in the second round is white?

Problem 3. John comes to a bus station in a random moment between 7:00 and 8:00 AM. A bus arrives at this bus stop also in a random moment between 7:00 and 8:00. If John arrives at the bus stop before 7:30, he waits 10 minutes and when bus is not arriving he walks to work. If he comes after 7:30 he waits 15 minutes and only then decides to walk.

Last Monday John came to work by bus. What is the probability that he arrived at the bus stop before 7:30?

Problem 4. Alice and Susan agreed to meet at a student mensa but they did not agree at the precise time of the meeting. Let us assume that both arrive at the mensa in a random moment between 1:00 and 2:00 PM. Alice will wait for Susan for 15 minutes, while Susan will wait for Alice for 20 minutes.

Let M denote the event that two friends will meet, let S be the event that Susan came before Mary, and by A we denote the event that Alice arrived before 1:20PM. Find the probabilities $\mathbb{P}(M)$, $\mathbb{P}(S)$, $\mathbb{P}(A)$, and $\mathbb{P}(A|M)$. Are the events A and M are independent?

Problem 5. An urn contains 2 white and 3 black balls. We select from it 3 different balls. Let X be the difference between the number of black balls and the number of white balls we selected.

Find the pmf, the CDF, and the expectation of X.