Introduction to Probability Theory
1st week: classical definition of probability

$\Omega = \{\omega_i : i \in I\}$, the set of possible outcomes of some random experiment, is called a sample space and any $A \subseteq \Omega$ is an event (sometimes ω_i is called an elementary event, $i \in I$). In the classical definition of probability we put

$$\mathbb{P}(A) = \frac{|A|}{|\Omega|}$$

Two useful properties of $\mathbb{P}(\cdot)$: for every $A, B \subseteq \Omega$ we have

$$\mathbb{P}(A) = 1 - \mathbb{P}(A'), \quad \mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B).$$

Some counting formulae:

- $\binom{n}{k}$ counts k-elements subsets of an n-element set;
- n^k counts sequences of length k whose terms are taken from an n-element set;
- $(n)_k = n \cdot \ldots \cdot (n-k+1) = \frac{n!}{(n-k)!}$ counts sequences of length k whose terms are taken from an n-element set in which all terms are different;
- $n!$ is the number of permutations of an n–element set.

Problems

Problem 1. Let us build a random word (possibly meaningless) of length 7 using letters a, b, c, d, e, f, g, h, i, j, k, l, m, n. Find the probability that all its letters will be different.

Problem 2. Find the probability that tossing a die n times we get

(a) six exactly three times;
(b) six at least three times.

Problem 3. Larry and Harry must decide who will pay for their lunch. Larry has got four straws, two long ones and two short ones. After a short discussion they decided that Harry will pick randomly two of them. If among them there will be one short one and one long one, he will pay for lunch, otherwise it will go on Larry. What is the probability that it is Harry who sponsors their meal?

Problem 4. Each of bridge players gets 13 cards each. What is the probability that each of them has one ace?

Problem 5. Find the probability that in a well-shuffled deck all four aces are placed next to each other.

Problem 6. From 52 cards we randomly select five. Find the probability that we will have: (a) full house; (b) one pair; (c) two pairs.

Problem 7. We randomly place k ball into n different urns. Find the probability that:

(a) each urn contains at most on ball (provided $k \leqslant n$)?
(b) in the first urn we have at most 2 balls?
(c) in the last urn we have at least 2 balls?
(d) none of the first two urns is empty?
(e) either the first urn, or the last one is empty?