<u>**Probabilistic Methods**</u>

**Homework #8**

Due: *Thursday, December 19th*

**Problem 9**

Let $p = n^{-\alpha}$ for some constant $\alpha \in (0, 1)$. Show that aas for every subset of vertices $S$ of $G(n, p)$ of size at most $(1-\alpha)n^{\alpha}(\ln n - 3 \ln \ln n)$, there exists a vertex $v \notin S$ which is adjacent to no vertices from $S$. From this fact deduce that aas $G(n, p)$ contains an independent set of size at least $(1 - \alpha)n^{\alpha}(\ln n - 3 \ln \ln n)$.

**Problem 10**

Let $p = n^{-\alpha}$ for some constant $\alpha \in (0, 1)$. Show that aas $G(n, p)$ contains an induced tree of size at least $(1 - \alpha)n^{\alpha}(\ln n - 3 \ln \ln n)$, i.e. $G(n, p)$ contains a connected subgraph on $(1 - \alpha)n^{\alpha}(\ln n - 3 \ln \ln n)$ vertices which contains no cycles.

*Remark* Each tree contains a vertex of degree one.