<u>**Probabilistic Methods**</u>

Homework #7

Due: *Thursday, December 5th*

Problem 8

We want to determine the size of the largest independent set in $G(n,p)$, i.e. to estimate $\alpha(G(n,p))$. During the class we have shown that it can be determined up to the factor $1+o(1)$ (or even better) when $p=1/2$ using just Chebyshev's inequality. On the other hand, it is not hard to show that when $p=c/n$ for some constant $c>0$, then the variance of the random variable X which counts independent sets of maximum size is much larger than $(\mathbb{E}X)^2$, so we cannot use Chebyshev's inequality to show that such sets exists. Find the largest value of $\alpha>0$ such that for every $\epsilon>0$ and $p=n^{-\alpha}$ one can show that $G(n,p)$ contains a set of size $(2-\epsilon)\frac{\ln(np)}{p}$ using Chebyshev's inequality.