Problem 3

Prove that there is an absolute constant $c > 0$ with the following property. Let A be an $n \times n$ matrix with pairwise distinct entries. Then there exists a permutation of the rows of A so that in the permuted matrix no column contains an increasing subsequence of length at least $c\sqrt{n}$.

Hint: Use the inequality

$$\binom{r}{s} \le \left(\frac{er}{s}\right)^s$$

and Stirling's formula

$$r! = (1 + o(1))\sqrt{2\pi r}\left(\frac{r}{e}\right)^r.$$