Probabilistic Methods
Homework #1
Due: *Thursday, October 17*

Problem 1

Let $\mathcal{F}$ be a family of n-element subsets containing fewer than $4^{n-1}/3^n$ sets, where $n \geq 4$. Prove that one can colour elements of these sets with **four** colours such that each of the colours appears in each set from $\mathcal{F}$.

Problem 2

Let k and n, where $k \leq n$, be natural numbers. Find an upper bound for the smallest number $m = m(k, n)$ for which there exists a colouring of $\{1, 2, \ldots, n\}$ with m colours such that no non-trivial arithmetic progression of length k is monochromatic.